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**US ARMY
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DERIVATION OF EXACT FORMULAS FOR CORRECTING
RADAR ANGLE DATA FOR ERRORS CAUSED BY
NONPERPENDICULARITY OF THE RADAR AXES

BY

WILLIAM L. SHEPHERD

USA ERDA-12

MARCH 1963

**WHITE SANDS MISSILE RANGE
NEW MEXICO**

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INSTRUMENTATION DEPARTMENT
U. S. ARMY ELECTRONICS RESEARCH AND DEVELOPMENT ACTIVITY
WHITE SANDS MISSILE RANGE
NEW MEXICO

A B S T R A C T

Exact formulas are derived for the correction of radar azimuth and elevation due to nonperpendicularity of the beam axis and azimuth axis to the elevation axis. Certain approximations are then obtained, and compared to the exact formulas. A method of combining these operations with coordinate transformations is given.

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INTRODUCTION

In this report we derive exact formulas for the correction of radar azimuth and elevation due to nonperpendicularity of the beam axis and of the azimuth axis to the elevation axis (Appendix A). Certain approximations are then obtained, and compared to the exact formulas. A method of combining these operations with coordinate transformations is given.

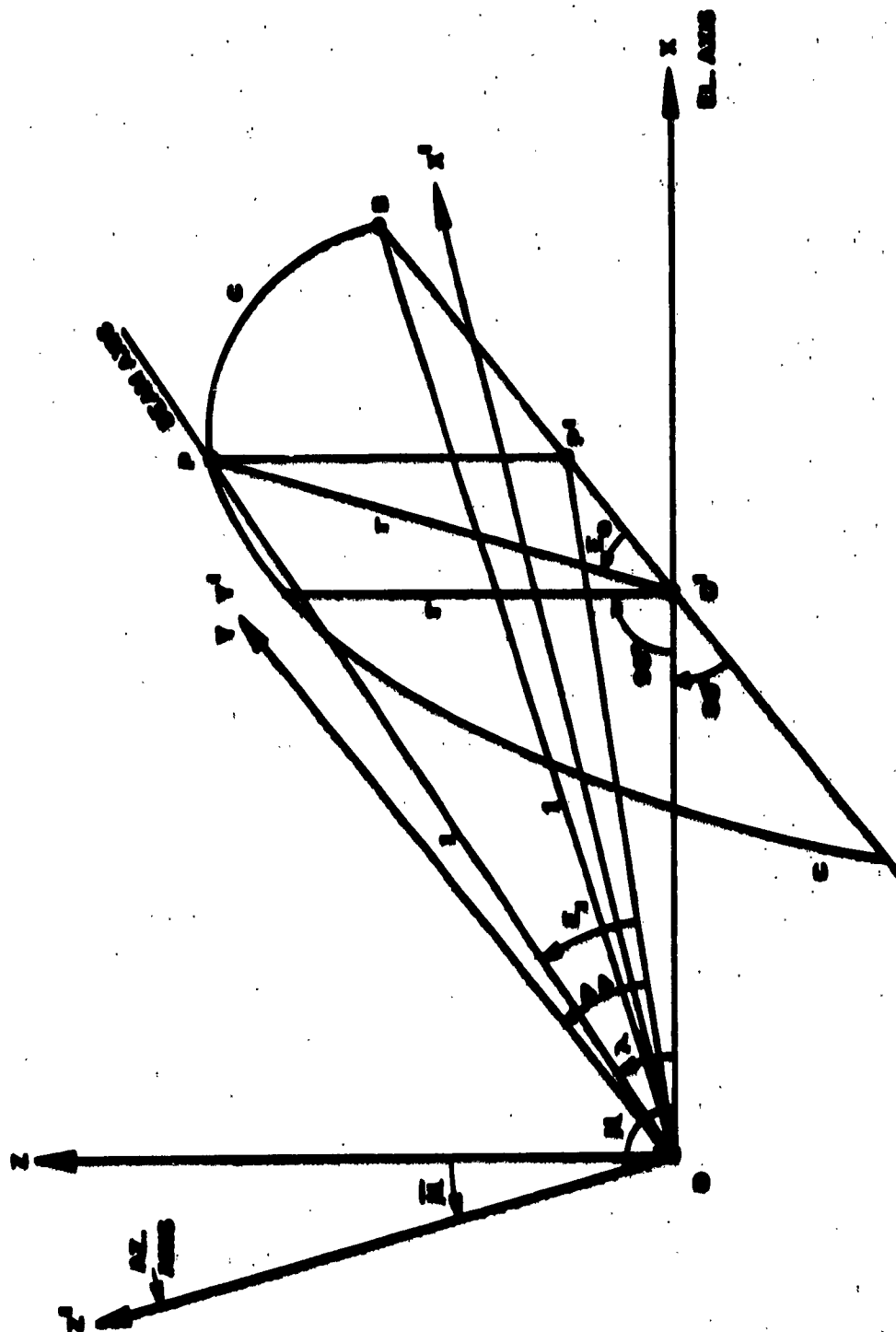
DEFINITION OF COORDINATE SYSTEMS

Assume that the azimuth axis, the elevation axis, and the beam axis intersect in a point, O (Figure 1). Let the ray OY be perpendicular to the plane determined by the azimuth axis and the elevation axis, and point north when the azimuth reading is zero. OY is in the plane through O perpendicular to the azimuth axis and the plane through O perpendicular to the elevation axis. Let OX denote the ray from O along the elevation axis which points east when OY points north. Let OZ be the ray from O so that $OXYZ$ is a right-hand orthogonal coordinate system. Notice that OZ is in the plane of the elevation axis and the azimuth axis and in the plane through O perpendicular to the elevation axis.

Let OZ' be the ray from O up along the azimuth axis, OY' coincide with OY and OX' be such that $OX'Y'Z'$ is a right-hand orthogonal coordinate system. Notice that OX' is in the XZ plane. If the azimuth axis and elevation axis are perpendicular, $OXYZ$ and $OX'Y'Z'$ coincide. In any event the azimuth of an object tracked by the radar as measured by sampling the azimuth data shaft position is a measure of the angle in the $X'Y'$ plane from north (the direction of OY when azimuth reads 0) to OY . We suppose this measurement to be corrected for azimuth data shaft eccentricities.

DERIVATIONS

Let OP denote the unit vector from O along the beam axis, and the positive sense of rotation of the elevation axis be determined by the right-hand rule. As the elevation axis rotates P generates an arc, c , of a circle whose center, O' , is on the elevation axis and whose plane is perpendicular to the elevation axis. $O'P$ generates the angle through which the elevation axis rotates. Let B be the point



occupied by P when the beam axis is in the first quadrant of the XY plane, and denote by E_0 the angle read by the elevation data shaft mechanism, supposed corrected for errors due to elevation data shaft eccentricities and to read zero when P coincides with B.

Let P' denote the perpendicular projection of P into O'B, and E_1 , the angle from OP' to OP. E_1 is the elevation of OP with respect to the XY plane.

Denote the angle from the elevation axis to the beam axis by λ , the angle from the elevation axis to the azimuth axis by μ , and consider the case $0 < \lambda \leq \frac{\pi}{2}$, $\pi > \mu \geq \frac{\pi}{2}$, and P in the first quadrant of the XY plane. This is one of four possible cases.

From the Figure, we see that r, the radius of c, is given by

$$r = \sin \lambda = \cos \bar{\lambda}, \quad (1)$$

$$\text{where } \bar{\lambda} = \frac{\pi}{2} - \lambda. \quad (2)$$

$\bar{\lambda}$, the angle between the beam axis and the XY plane, is the nonperpendicularity of the beam axis and the elevation axis.

Then

$$\sin E_1 = P'P = r \sin E_0 = \cos \bar{\lambda} \sin E_0, \quad (3)$$

and

$$X = OO' = \cos \lambda = \sin \bar{\lambda}, \quad (4)$$

$$Y = O'P' = r \cos E_0 = \cos \bar{\lambda} \cos E_0, \quad (5)$$

$$Z = P'P = r \sin E_0 = \cos \bar{\lambda} \sin E_0, \quad (6)$$

where X, Y, Z denote the coordinates of P with respect to OXYZ.

Now OXYZ can be brought into coincidence with OX'Y'Z' by the right-hand rotation about OY through the angle $2\pi - \mu$, where

$$\bar{\mu} = \mu - \frac{\pi}{2}. \quad (7)$$

$\bar{\mu}$, the angle between the azimuth axis and the XY plane, is the nonperpendicularity of the azimuth axis and elevation axis.

Hence, with X' , Y' , Z' denoting the coordinates of P with respect to $OX'Y'Z'$, and setting

$$A = \begin{bmatrix} \cos(2\pi - \bar{\mu}) & 0 & -\sin(2\pi - \bar{\mu}) \\ 0 & 1 & 0 \\ \sin(2\pi - \bar{\mu}) & 0 & \cos(2\pi - \bar{\mu}) \end{bmatrix}$$

$$= \begin{bmatrix} \cos \bar{\mu} & 0 & \sin \bar{\mu} \\ 0 & 1 & 0 \\ -\sin \bar{\mu} & 0 & \cos \bar{\mu} \end{bmatrix}, \quad (8)$$

we have

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = A \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (9)$$

Equations (4), (5), (6), (8), and (9) together yield

$$X' = \cos \bar{\mu} \sin \bar{\lambda} + \sin \bar{\mu} \cos \bar{\lambda} \sin E_0 \quad (10)$$

$$Y' = \cos \bar{\lambda} \cos E_0 \quad (11)$$

$$Z' = -\sin \bar{\mu} \sin \bar{\lambda} + \cos \bar{\mu} \cos \bar{\lambda} \sin E_0 \quad (12)$$

Let A_0 denote the azimuth of OY' , as read from the azimuth data shaft mechanism supposed corrected for errors due to azimuth data shaft eccentricities, and to read zero when OY' is in the north reference direction of the radar. Let ΔA denote the azimuth of OP with respect to OY' in the $X'Y'$ plane, A_2 the azimuth of OP referred to the initial ray in the $X'Y'$ plane from which the azimuth data shaft reads azimuth, and E_2 the elevation of OP with respect to the $X'Y'$ plane.

Then

$$\sin E_2 = Z' = \cos \bar{\mu} \cos \bar{\lambda} \sin E_0 - \sin \bar{\mu} \sin \bar{\lambda}, \quad (13)$$

$$\tan \Delta A = \frac{X'}{Y'} = \frac{\cos \bar{\mu} \sin \bar{\lambda} + \sin \bar{\mu} \cos \bar{\lambda} \sin E_0}{\cos \bar{\lambda} \cos E_0}$$

$$= \frac{\cos \bar{\mu} \tan \bar{\lambda} + \sin \bar{\mu} \sin E_0}{\cos E_0} \quad (14)$$

and

$$A_2 = A_0 + \Delta A = A_0 + \tan^{-1} \left(\frac{\cos \bar{\mu} \tan \bar{\lambda} + \sin \bar{\mu} \sin E_0}{\cos E_0} \right), \quad (15)$$

$$E_2 = \sin^{-1} \left(\cos \bar{\mu} \cos \bar{\lambda} \sin E_0 - \sin \bar{\mu} \sin \bar{\lambda} \right). \quad (16)$$

Equations (15) and (16) are exact. For them to be most useful, $\bar{\lambda}$ and $\bar{\mu}$ must be constant and reading errors must be correctible. In practice $\bar{\lambda}$ and $\bar{\mu}$ are constant.

For small $|\bar{\lambda}|$ and $|\bar{\mu}|$, $|\sin \bar{\mu} \sin \bar{\lambda}|$ is very small. Referring to (16), we see that

$$E_2 \approx \sin^{-1} \left[\cos \bar{\mu} \cos \bar{\lambda} \sin E_0 \right] \quad (17)$$

is a good approximation, but $\cos \bar{\mu} \cos \bar{\lambda}$ and $\sin \bar{\mu} \sin \bar{\lambda}$ are constant, so the computation required in (17) is not appreciably shorter than the computation required in (16).

Referring to (14), we see that, for small E_0 , $\sin \bar{\mu} \sin E_0$ is small and

$$\tan \Delta A \approx \cos \bar{\mu} \tan \bar{\lambda} \sec E_0 \approx \bar{\lambda} \cos \bar{\mu} \sec E_0 \approx \bar{\lambda} \sec E_0 \approx \Delta A \quad (18)$$

if we successively use

$$\sin \bar{\mu} \sin E_0 \approx 0, \tan \bar{\lambda} \approx \bar{\lambda}, \cos \bar{\mu} = 1, \tan \Delta A = \Delta A.$$

Again the computing time saved by such approximation is not great, especially if one checks the magnitude of E_0 to decide if the approximations are sufficiently close to be useful.

For some radars, $\bar{\mu}$ may be negligible and

$$\tan \Delta A = \tan \bar{\lambda} \sec E_0.$$

In any case, $\bar{\lambda}$ and $\bar{\mu}$ are constant and little computing time is saved by approximating the exact expressions.

CONVERSION TO ANOTHER COORDINATE SYSTEM

In case the desired output of the radar is the set of cartesian coordinates of a missile referred to some coordinate system other than $OX'Y'Z'$ one does not need equations (15) and (16). Suppose, for example, that we wish to refer the missile position to a rectangular coordinate system $OX''Y''Z''$ with OZ'' along the geodetic vertical, OY'' in some specified north direction and OX'' such as to complete a right-hand coordinate system. Let R, A, E denote the measured range, azimuth, and elevation of the missile, and B the matrix of the coordinate transformation from $OX'Y'Z'$ to $OX''Y''Z''$.

Then

$$\begin{vmatrix} X'' \\ Y'' \\ Z'' \end{vmatrix} = C \begin{vmatrix} RX \\ RY \\ RZ \end{vmatrix} \quad (19)$$

where $C = BA$.

CONCLUSIONS

Approximate formulas for computing corrections to azimuth and elevation should be used instead of the exact formulas only when rapid computation is extremely important.

For some common uses of radar data, explicit computation of azimuth and elevation is redundancy.

A P P E N D I X A

WORKING DEFINITIONS

1. Azimuth Axis - The axis about which the radar antenna mount rotates as azimuth varies. (Should be vertical.)
2. Beam Axis - Direction of the beam of energy incident upon the receiving antenna.
3. Elevation Axis - The axis about which the antenna rotates as elevation varies. (Should be horizontal.)

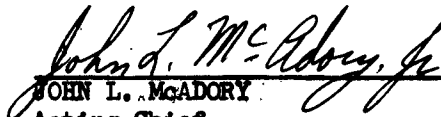
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FOR THE COMMANDER:


L. W. ALERO
Major, AGC
Adjutant

AD ACCESSION NR Instrumentation Department, U. S. Army Research and Development Activity, White Sands Missile Range, New Mexico DERIVATION OF EXACT FORMULAS FOR CORRECTING RADAR ANGLE DATA FOR ERRORS CAUSED BY NONPERPENDICULARITY OF THE RADAR AXES, By William L. Shepherd, Technical Report USA RMA-12, 7 pp incl illus, March 1963. <u>UNCLASSIFIED Report</u> Exact formulas are derived for the correction of radar azimuth and elevation due to nonperpendicularity of the beam axis and azimuth axis to the elevation axis. Certain approximations are then obtained, and compared to the exact formulas. A method of combining these operations with coordinate transformations is given.	UNCLASSIFIED 1. Radar 2. Measurement 3. Geometry 4. Data Processing Copies of this document may be obtained upon request from: Armed Services Technical Information Agency Arlington Hall Station ARMY: AFPR Arlington 12, Virginia	AD ACCESSION NR Instrumentation Department, U. S. Army Research and Development Activity, White Sands Missile Range, New Mexico DERIVATION OF EXACT FORMULAS FOR CORRECTING RADAR ANGLE DATA FOR ERRORS CAUSED BY NONPERPENDICULARITY OF THE RADAR AXES, By William L. Shepherd, Technical Report USA RMA-12, 7 pp incl illus, March 1963. <u>UNCLASSIFIED Report</u> Exact formulas are derived for the correction of radar azimuth and elevation due to nonperpendicularity of the beam axis and azimuth axis to the elevation axis. Certain approximations are then obtained, and compared to the exact formulas. A method of combining these operations with coordinate transformations is given.	AD ACCESSION NR Instrumentation Department, U. S. Army Research and Development Activity, White Sands Missile Range, New Mexico DERIVATION OF EXACT FORMULAS FOR CORRECTING RADAR ANGLE DATA FOR ERRORS CAUSED BY NONPERPENDICULARITY OF THE RADAR AXES, By William L. Shepherd, Technical Report USA RMA-12, 7 pp incl illus, March 1963. <u>UNCLASSIFIED Report</u> Exact formulas are derived for the correction of radar azimuth and elevation due to nonperpendicularity of the beam axis and azimuth axis to the elevation axis. Certain approximations are then obtained, and compared to the exact formulas. A method of combining these operations with coordinate transformations is given.	UNCLASSIFIED 1. Radar 2. Measurement 3. Geometry 4. Data Processing Copies of this document may be obtained upon request from: Armed Services Technical Information Agency Arlington Hall Station ARMY: AFPR Arlington 12, Virginia	UNCLASSIFIED 1. Radar 2. Measurement 3. Geometry 4. Data Processing Copies of this document may be obtained upon request from: Armed Services Technical Information Agency Arlington Hall Station ARMY: AFPR Arlington 12, Virginia
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